Students play commercial games of strategy, which illuminate their reasoning and thinking. Isn't it time your students spent some time in math class playing games?

WE'RE IN MATH CLASS PLAYING GAMES, NOT PLAYING GAMES IN MATH CLASS

P. Janelle McFeetors and Kylie Palfy

The classroom is alive with chatter, laughter, and occasional groans. Students are intensely focused on the matter at hand: winning. Gobblet Gobblers, Tic Stac Toe™, Othello, and Go strategy games are set up at tables, on the floor, and in reading corners. To a casual observer, this might look like a social gathering in school, but it is not recess and not play time. There is an intensity to the reasoning in play. Students are forming conjectures, explaining moves, and justifying their strategies. Students described the scene in this way: "We're in math class playing games, not playing games in math class."

These early experiences of reasoning while playing games of strategy are foundational for future proofs that students will be expected to build using conventionally structured arguments. But how did game

playing in school occur? How can we be sure that mathematical reasoning is going on? We investigated these questions to understand how to improve students' reasoning.

Knowing that reasoning is a cornerstone of success in mathematics (CCSSI 2010; NCTM 2014), we believed that it was important to focus on logical thinking as a mathematical practice. We chose the context of abstract strategy games because games in this category depend on logical reasoning for players to win. In fact, the games require logic to move from singular moves to effective strategies. We also wanted students to value reasoning as an integral part of thinking mathematically, which could then be applied across all mathematics content.

We use Reid's (2002) definition of mathematical reasoning as an overall, systematic, and logical pattern of behavior. In this article, we share two pedagogical practices: setting the stage and purposeful prompting. The intention is to describe the pedagogic decisions that led to an environment in which reasoning developed through sustained engagement. We then illustrate the qualities of students' emergent reasoning as patterns of thinking to legitimize a focus on learning reasoning.

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SETTING THE STAGE

The creation of the educative environment was not accidental pedagogically. We collaborated with a teacher, Nancy, to explore how commercially available games could support emerging reasoning in her multi-aged grade 5 and 6 class. We chose games that were strategic and contained no element of chance. Players are required to develop and defend effective strategies to win. (See the **sidebar** on page 543 for game descriptions.)

The desire to determine how to win encouraged students to engage in the Standard for Mathematical Practice (SMP) 3: "Construct viable arguments and critique the reasoning of others" (CCSSI 2010, p. 6). The games provided opportunities for students to analyze specific moves, make conjectures about those moves in alternate situations, test those moves to convince themselves and others they worked, and generalize toward strategies that required justification and judgment. The veracity of the reasoned arguments was evaluated within the game play to decide on their defensibility.

Rather than simply exploring the games, Nancy set the expectation that expert, strategic players continually refine strategies based on inductive reasoning. Given specific board arrangements, the students then determine weaknesses and strengths of strategies. Owen, one of the students, recognized that as an expert he needed to show that he had "a good chance of winning and a lot of strategies" each time he played. However, we knew that eliciting reasoning would require more than just an engaging context. We also intentionally shifted students' focus to emphasize the strategic aspects of game play.

(PREVIOUS PAGE) NICK WHITE/THINKSTOCK

Even though these are two-player games, students initially played as partners against another pair of students. The structure was intentional because collaboration and discussion in a social context are integral to the thoughtful construction of reasoning (Graves 2011; Olson 2007). When partnerships became a matter of choice, Nancy reminded the students to "pick a mathematically productive partner—someone who challenges you to think hard and generate new ideas." As students played alongside each other, they collaboratively investigated strategies and explained their reasons for suggesting particular moves.

During game play, the collaboration between students and across opposing pairs was also encouraged through listening to and analyzing strategies being shared. As teachers and researchers, we modeled listening by sitting with groups of students attuned to the explanations given for moves. When we heard compelling reasoning, we wrote down students' comments to document their mathematical thinking. Nancy explained this as a powerful practice:

> We were copying down what they said so that they could see what we privileged as information and got excited about. And they started to think about why that was important, and why their ideas were meaningful. And then they started to build on that.

While we listened and documented their comments, students realized that we valued their reasoning. In turn, recognizing the value of students' moves as reasoned play encouraged them to develop more sophisticated reasoning and evaluate justifications. For example, Floyd asked his partner while playing Gobblet Gobblers, "Why do you use a triangle strategy?" Partner Sasha

Fig. 1 These prompts were meant to elicit reasoning.

Analyzing specific moves:
- Why is that a good opening move to make?
- Is it better to be the first or the second player? Why?

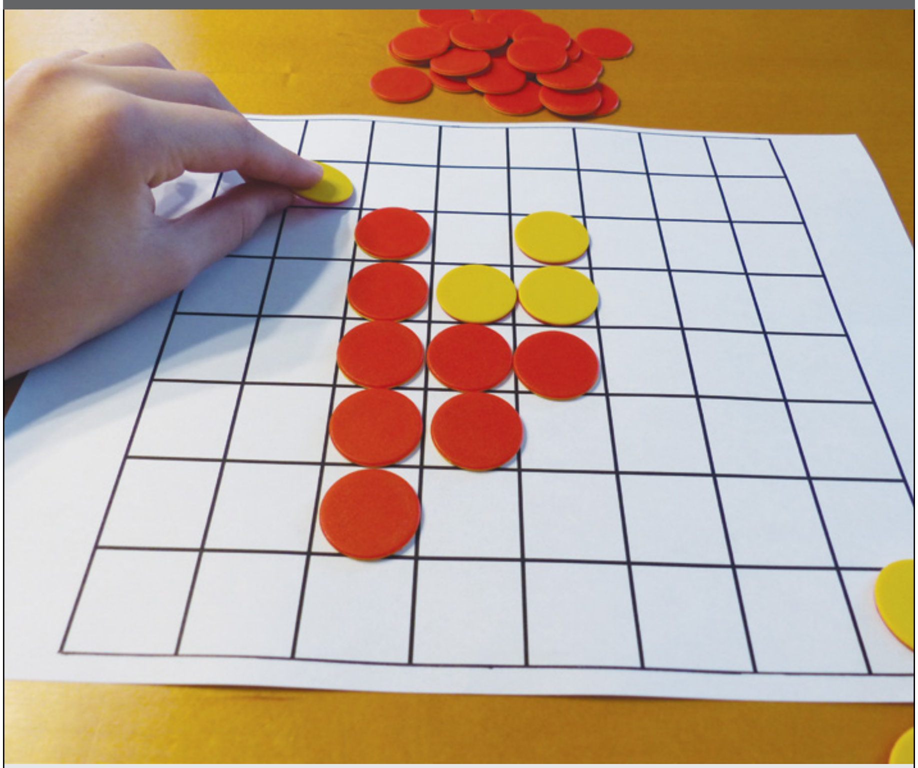
Representing moves:
- Do you spend more time trying out possible moves with a playing piece on the board, or just looking at the board?

Justifying moves:
- Can you explain why you made that move?
- What did you do so as not to give your opponent an advantage?
- How were you able to block your opponent from making a good move?

Generalizing toward strategies:
- What names did you make up for your moves or arrangements to help talk with your partner?
- At what point in the game was that useful?

Modifying strategies:
- What tic tac toe strategy helped you play Gobblet Gobblers? Tic Stac Toe?
- Did your strategy change while you were playing? How?

Fig. 2 Kurt made a convincing argument for a move while playing Othello.



"If we place here, they'd get a corner, so that's not good. You've got to look at the long run. . . . You have to fully look at it. You might get some now but then lose them."

responded, "Because it gives me more options." Floyd's question resulted in Sasha explaining that the move produced multiple ways to win. The offensive gain indicated an important strategy similar to tic tac toe as Floyd accepted Sasha's reason for her placement of pieces. Students saw each other's strategies and defenses as valuable and were afforded the opportunity to develop reasoning within an authentic context of playing a game.

PURPOSEFUL PROMPTING

While students readily engaged in exploring the games to analyze specific moves, we found that to sustain productive mathematical thinking we needed to prompt for reasoning that had led to winning strategies. Building on the instructional practice to "pose purposeful questions" (NCTM 2014, p. 10), we used verbal and written prompts to encourage students to express their reasoning as a second powerful practice.

While students were playing the games, we posed questions aloud. We found that it was helpful to have some questions prepared in advance to elicit emergent reasoning, but to be responsive to how each game unfolded. (See **fig. 1** for examples of teacher-posed prompts.) These queries also allowed us to ask follow-up questions immediately to probe for more sophisticated reasoning.

Nancy explained that the prepared questions enabled her to "see another question that was forming to do with strategy, like, 'I noticed you just did this and I'm wondering what the purpose was.' " **Figure 2** depicts how Kurt's sophisticated strategy considered several moves in advance like an informal game tree of future moves, including a deductive reasoning structure of an if-then statement. Nancy's prompt encouraged Kurt and his classmates to extend an explanation of a specific move to articulating

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a strategy and its justification by explaining the purpose of the move. The questions were catalysts for deeper engagement with the games and improved reasoning. One student, Esme, commented, “The questions you asked me actually helped me play the game better. They made me think about the strategies.” We were thrilled that students also recognized the importance of teacher prompting.

Verbal prompting added richness to students’ reasoning; however, we thought it would be important for students to record their strategies and explain their reasoning with words and drawings. We developed record sheets to promote the idea that students had strategies and reasoning worth recording (see **fig. 3**’s sample prompts). In fact, in our final class, Nira—a student typically unengaged in class—showed so much pride in her thinking that she insisted that Nancy read her record sheet before leaving class.

As they played, students chose when to answer the questions. After play was complete, we allotted 10 minutes for them to consolidate their responses. The record sheets gave students a chance to reflect on the strategies and reasoning that they had developed during play. **Figure 4** shows Lelanie’s strategies while playing Othello. Her emergent reasoning was evidence of an effective Othello strategy of securing corner locations, with possible growth toward more sophisticated strategic thinking like Kurt’s (see **fig. 2**) in being able to see several moves in advance to set up securing the corners. The use of the word “always” in Tip 1 demonstrated how multiple experiences with varied game configurations helped her to form a generalization, which was evidence of inductive reasoning.

Fig. 3 Prompts for specific play situations were meant to elicit information for recording game strategies and reasoning.

Prompts for convincing:

- For each person, tell me what you said to your partner to convince him [or her] of a good move at any point in the game.
- Strategic players often consider more than just the move they are making. What else are you thinking about during a turn, besides just the piece you are going to put down?

Prompts for generalizing:

- Tell me how you plan ahead either to set up a win for you or a loss for your opponent.
- You have been hired to make a tips and tricks website for [game]. Create two important tips to help players.

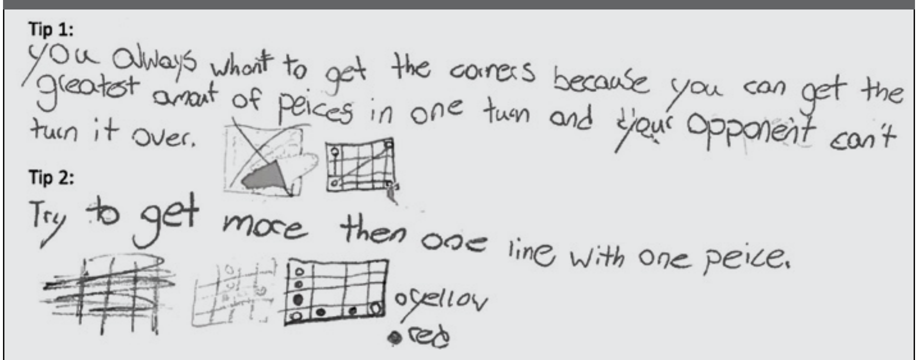
Prompts for justifying:

- Look at the three boards, showing different early game moves for [game]. Rank how good the move was by writing great, okay, or poor. Defend your ranking.
- In all the games, students told me about playing on the edges or corners. Can you tell me how playing on the edges and/or corners helped in [game]?

Prompts for analyzing:

- Mathematicians who research game theory are interested in all the different moves in a game. Can you draw all the opening moves for three related games: tic tac toe, Gobblet Gobblers, and Tic Stac Toe? Give a count and reason for the count.
- This board is a few turns into the game. You’re the [piece color] player. Draw where you would put your next piece. Why is it a good move?

Fig. 4 Lelanie spelled out her Othello strategies.



Students began to take ownership of their strategic game play and reasoning. During the final class, Grace caught our attention, being ready with pencil and notebook in hand. As she watched her friends play Gobblet Gobblers, we heard her ask, “Okay! So what did you think about before you made your move?” She recorded the strategies that Nira and Alice explicated, without being asked to do so (see **fig. 5**). We noticed that these students’ analyses of where to start playing led to their decision of beginning in the corner and moving to the middle. Although recording the strategy without justification could be viewed as emergent reasoning, the girls' metacognitive awareness of the shift in strategy signified strategic thinking. The students knew what they were doing was important; their actions were evidence of the effectiveness of the prompts.

As students were asked to explain and evaluate their strategies and synthesize the knowledge gained across many different game configurations (Chapin, O'Connor, and Anderson 2009), the questions elicited analysis that led to strings of if-then reasoning. Alison described possibilities for her next move in Othello:

> After Patricia moved, I have different angles. So I could go this way if I put it here, and if Patricia wouldn't have moved there I could have gone this way. I didn't really consider going that way though, I was just kind of thinking about going that way. But if I would have moved here I would have been able to go that way, or a diagonal.

When the teacher prompted Alison to explain her defense of a move, it caused Alison to evaluate possible moves based on Patricia's choices that were not at first apparent. This demonstrated a growing sophistication in reasoning because we found that students just beginning to play Othello had limited visualizations of possible moves. Purposeful prompting led students through a progression of thinking that advanced from the specific to general, from simple explorations to emergent reasoning, and from tentative conjectures to confident justifications. This evolution was observable as a larger pattern of mathematical thinking.

REASONING AS PATTERNS OF THINKING

The educative context was set, and we were purposefully prompting students. But how could we tell that what they were engaging in was reasoning? Often reasoning is categorized by its form, like inductive or deductive, which is a helpful way to evaluate arguments recorded on paper. However, we found that the students' emergent reasoning was more dynamic and usually richer when expressed aloud. This realization meant that we needed to listen to more than just the final product.

Fig. 5 Grace, Alice, and Nira recorded their evolving view of playing Gobblet Gobblers.

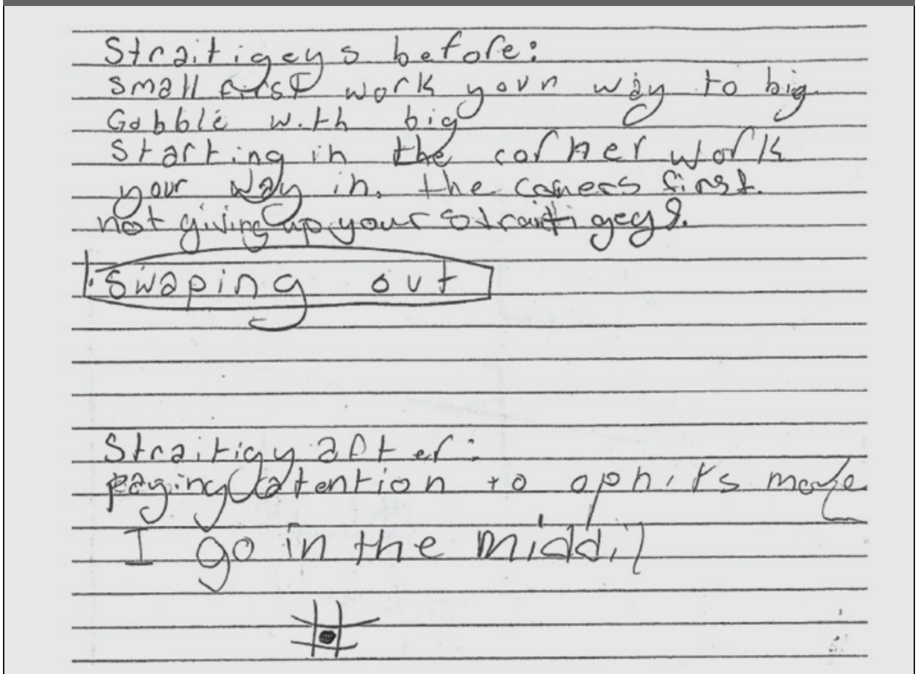
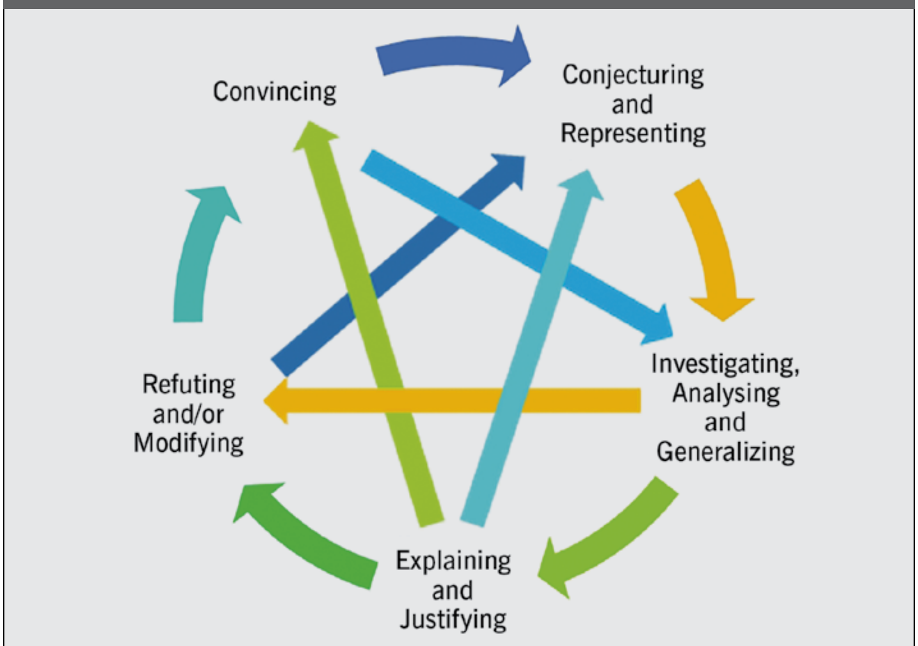


Fig. 6 The verbs of mathematical reasoning both intersect and move in a circular manner.



Our close attention enabled us to notice students' reasoning in action, as a process that grew out of a context that required logical thinking.

As we observed students during game play, we saw such reasoning processes as specializing, conjecturing, representing, generalizing, investigating, analysing, explaining, justifying, refuting, modifying and convincing (Lannin, Ellis, and Elliott 2011; Mason, Burton, and Stacey 2010; Mueller and Maher 2009) as creating a logical pattern of behavior. The diagram we created (see **fig. 6**) depicts our interpretation of the interconnectedness of students' actions in this pattern, which we call the verbs of mathematical reasoning. The arrows indicate a primarily circular flow and summarize common pathways that students traversed in defending moves and strategies.

Examples of these verbs being enacted existed all around the classroom. We offer four vignettes of students' emergent reasoning as examples of the quality of their thinking. As we became more adept at noticing students' reasoning through these verbs, their constant defense with plausible and evolving reasoning convinced us we had found an engaging context that provoked reasoning. We have drawn attention to the verbs with italics.

Fig. 7 Renée's emergent strategies for Go involved territorial considerations.

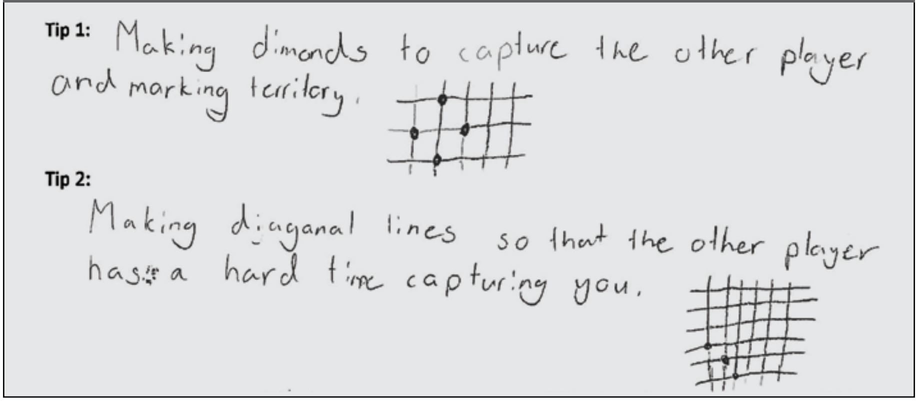


Fig. 8 Eric's modification was based on a classmate's strategy.

Eric: Mh-mm! I got—I have Floyd in the checkmate position [laughter].
Interviewer: So can you show me which pieces those are—the checkmate position?
Floyd: 'Cause this, it was like this and I was here.
Interviewer: Okay, so how does the checkmate position help you?
Floyd: Because then if I put it here he can just go here. If I put it here, he goes here. And if I put it over here then he can go there.
Interviewer: So you [Eric] have set up two ways of winning?

Both Eric and Floyd agree.

Vignette 1

After many rounds of Go, Renée identified emerging strategies and *explained* when and how to use them. Her *conjecturing* focused on board configurations, so she used her play to *investigate* arrangements that seemed to lead to wins. She *justified* her ideal board configurations of diamonds and diagonal lines by *generalizing* the experiences she had during matches (see **fig. 7**). Tip 2 is an important strategy for expanding territory while playing Go. Renée demonstrates inductive reasoning, using many instances to state a general strategy.

Vignette 2

While playing Gobblet Gobblers, Floyd *explained* a self-named "checkmate" position to ensure success based on *analyzing* previous wins. His reasoning by analogy—using a term he was familiar with from chess—was offered to *refute* a claim made by Eric: "It's best to have two big gobblers in the corners." At the time, Eric seemed unsure of how Floyd's checkmate strategy worked, stating, "I don't know how you'd get that." Later in an interview, Eric *modified* his play and incorporated his partner's move (see **fig. 8**), a sign of a *convincing* justification.

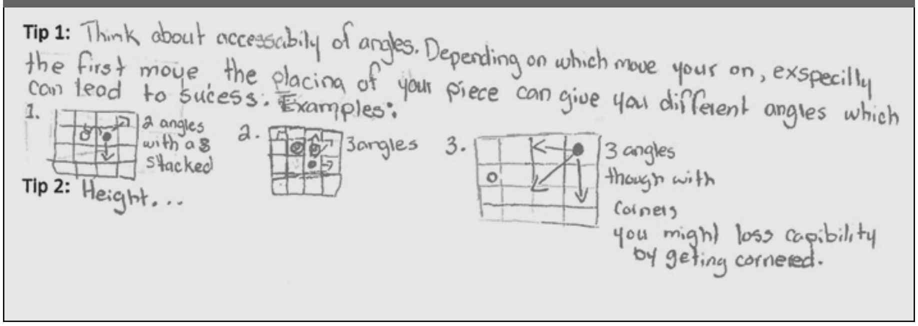
Vignette 3

During Alison's first few rounds of Tic Stac Toe, teacher prompting led to her *conjecture* that the first play needed to set up different ways of winning. She *investigated* by trying

The students knew that what they were doing was important; their actions were evidence of the effectivess of the prompts.

different starting places. By *representing* the board in a pictoral form (in **fig. 9**) she was able to *analyze* starting positions based on her prediction of subsequent moves and the likelihood of moves leading to a win. The drawings demonstrate early engagement in imagistic reasoning that is foundational to more comprehensive analysis later on. We noticed Alison applying another mathematical concept of angles to be *convincing* in tip 1. We viewed this as an emergent strategy because we anticipated that Alison could be prompted to expand her analysis in tip 2 by considering the different angles created by stacking pieces.

Fig. 9 Allison's emergent strategies for Tic Stac Toe leaned toward geometry.



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Vignette 4

Eve's logical thinking moved beyond the immediacy of the game to *analyzing* a global strategic approach. She paused an Othello game long enough to *explain* how she played against her opponent: "If you were playing someone experienced, you'd have to shift strategies so they couldn't pick up your strategy, but someone new wouldn't know necessarily what you are doing. Shifting strategies becomes important as you get good. You have to have a plan C and plan D." Eve *represented* her argument with an if-then structure and continued to *justify* developing different action plans to *modify* her play based on her opponent's moves. These are early demonstrations of a shift to conventionally structured arguments required in future proving.

Eve demonstrated skillfulness at adaptive reasoning, defined as the "capacity for logical thought, reflection, explanation, and justification" (Kilpatrick, Swafford, and Findell 2001, p. 5). The importance of adaptive reasoning, "the glue that holds everything together, the lodestar that guides learning" (p. 130), cannot be overstated. Student interactions with the dynamic context of game play required them to adjust and modify strategies and thus develop reasoning that was adaptive.

DISCUSSION

When chosen carefully, framed as educative experiences (Dewey 1938/1997), and scaffolded with purposeful questions, strategy games are an authentic context

Examples of Strategy Games

These games were used with grade 5 and 6 students:

Gobblet Gobblers: A variation of tic-tac-toe. Each player has two sets of small, medium, and large gobblers, or playing pieces. Players place a piece in an empty location or on top of a gobbler to hide it on the board. A three-in-a-row situation creates a win.

Tic Stac Toe: A variation of tic-tac-toe. The board is a 4 × 4 grid, and pieces can be stacked on top of one another up to four layers high. A four-in-a-row creates a win.

Othello: Also known as Reversi. Players place a disc on spaces on an 8 × 8 board to surround and flip opponent's pieces in horizontal, vertical, and diagonal lines to gain territory. The player with the most pieces on the board wins.

Go: Also known as Weiqui or Baduk. This is the most difficult logical reasoning game. It is played on a 19 × 19 board. Players place stones on intersections to gain territory and to remove opponent's pieces. The highest points win the game.

within which students can develop mathematical reasoning. Initially, students played the games with little justification for decisions made. However, after gaining familiarity with the games and through the provocation of prompts, they generated emergent strategies. As play continued, the students engaged in the verbs of mathematical reasoning. They began to provide justifications, negotiate terms, and defend strategies. Since reasoning is named in three of the eight SMPs (CCSSI 2010) and is more strongly correlated to success in mathematics than arithmetic ability (Nunes et al. 2012), it is clear that mathematics classrooms should include activities that specifically target the development of this mathematical process.

Our hope in integrating abstract strategy games in mathematics class was that students would develop ways of reasoning logically about the strategies they were creating. One of the surprises was how quickly they improved in their reasoning. This was heard as students' comments became increasingly convincing statements and their game play increasingly effective. Using the verbs of mathematical reasoning helped us identify many moments that comprised students' growth in reasoning. Isn't it time your students spent some time in mathematics class playing games?

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Janelle McFeetors, janelle.mcfeetors@ualberta.ca, is an assistant professor of mathematics education at the University of Alberta in Edmonton. She is interested in students’ development of mathematical processes and how to support parents’ engagement in their children’s mathematics learning. **Kylie Palfy**, kylie.palfy@spschools.org, is a numeracy lead teacher who also teaches grade 2 French Immersion at Muriel Martin School in St. Albert, Alberta. She is interested in helping teachers develop their students’ problem-solving and reasoning skills.